$F''_{01}(0) = -0{\cdot}173879, \quad \Phi_{01}(0) = -0{\cdot}226844,$

$F''_{02}(0) = 0{\cdot}134655, \quad \Phi_{02}(0) = 0{\cdot}030392,$

$G_{00}(0) = 0{\cdot}0, \quad G'_{00}(0) = 1, \quad G''_{00}(0) = -1{\cdot}837319,$

$G_{01}(0) = -0{\cdot}300701, \quad G'_{01}(0) = 0{\cdot}732916,$

$G''_{01}(0) = -1{\cdot}399446, \quad G_{02}(0) = -0{\cdot}311263,$

$G'_{02}(0) = 0{\cdot}804204, \quad G''_{02}(0) = -1{\cdot}578787,$

$\Phi_{10}(0) = -0{\cdot}118237, \quad \Phi_{11}(0) = 0{\cdot}077670,$

$\Phi_{12}(0) = -0{\cdot}094576.$

CONCLUSIONS

The effect of viscous dissipation in the case of an assigned surface heat-flux is to make necessary a larger difference between t_w and t_∞ for the covection of a heat-flux to the fluid. The surface temperature is given by the relationship

$$t_w - t_\infty = -\frac{q''x/k}{(\sigma Gr_x/5)^{\frac{1}{5}}}$$

$$\times [1{\cdot}147565 + 0{\cdot}226844\sigma^{-\frac{1}{2}} - 0{\cdot}030392\sigma^{-1}] [1 + 5r\epsilon],$$

where

$$r = \frac{\phi_1(0)}{\phi_0(0)} = 0{\cdot}103033 - 0{\cdot}088049\sigma^{-\frac{1}{2}} + 0{\cdot}102548\sigma^{-1}.$$

The important ratio r has the values 0·085444, 0·095253, 0·100351 and 0·102163 for $\sigma = 10$, 10^2, 10^3 and 10^4 respectively. The value given by Gebhart [1] for $\sigma = 10^2$, namely 0·09547, compares very well with the corresponding value obtained above. However, though r increases with σ it has an asymptotic value 0·103033 for $\sigma = \infty$.

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PREDICTION OF FLOW AND HEAT TRANSFER IN TURBULENT CYLINDRICAL WALL JETS

S. V. PATANKAR* and A. K. SINGHAL†

Indian Institute of Technology, Kanpur, India

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NOMENCLATURE

- C, a curvature parameter ($\equiv d/S$);
- c_p, specific heat at constant pressure;
- d, diameter of the rod;
- h, local heat-transfer coefficient;
- K, a mixing-length constant;
- l, the mixing length;
- p, pressure;
- q, heat flux;
- r, distance from the axis of symmetry;
- r_I, radius of inner boundary of the wall jet (i.e. the radius of the rod);
- Re_S, a Reynolds number ($\equiv \rho u_S S/\mu$);
- S, slot height;

* Present address: Mechanical Engineering Department, Imperial College, London.

† Present address: Tata Consulting Engineers, Bombay.

St, a Stanton number $[\equiv h/(\rho c_p u_{max})]$;
T, temperature;
u, velocity in the x direction;
u_{max}, maximum velocity in the wall jet;
u_S, velocity at the slot;
v, velocity in the r direction;
x, the longitudinal distance;
δ, a boundary-layer thickness;
$\delta_{\frac{1}{2}}$, "half" thickness of the wall jet (i.e. the distance between the wall and the point in the outer region where $u = u_{max}/2$; see Fig. 1);
λ, a mixing-length constant;
μ, the laminar viscosity;
μ_{eff}, the effective viscosity;
ρ, density;
σ_{eff}, the effective Prandtl number;
τ, shear stress.

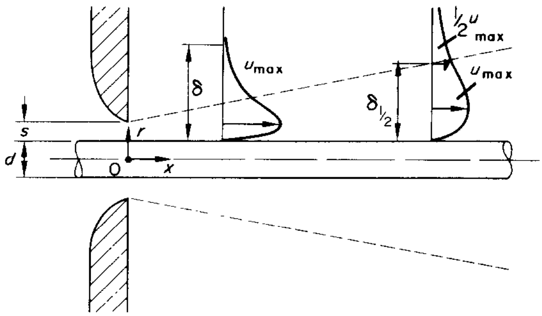

FIG 1. The cylindrical wall jet.

1. THE PURPOSE OF THE PRESENT COMMUNICATION

EXTENSIVE literature exists on the prediction of two-dimensional, turbulent, boundary-layer flows. At the present state of knowledge, a prediction procedure must employ a "turbulence model" which relates the turbulence quantities to the time-averaged properties of the flow. A simple turbulence model is the Prandtl mixing-length hypothesis, which has been used for a variety of boundary-layer flows (see e.g. [1, 2]) and found to be satisfactory in many cases. The present communication describes an application of this model to wall-jet flows having strong transverse curvature.

Figure 1 shows the flow situation considered. A jet emerges from an annular slot and flows longitudinally on a cylindrical rod. The resulting flow is a cylindrical wall jet and has appreciable transverse curvature when the slot height S is not small compared with the rod diameter d. Starr and Sparrow [3] and Manian *et al.* [4] have reported flow and

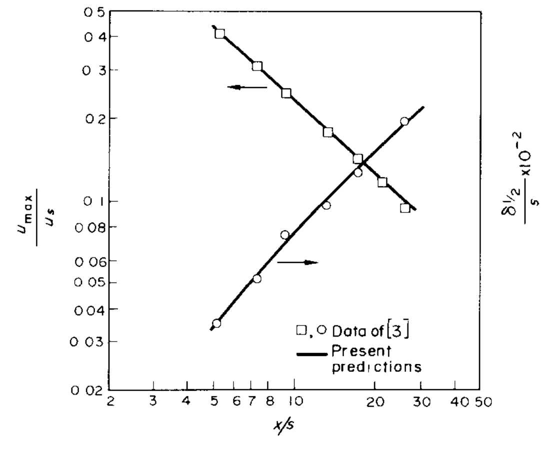

FIG. 2. Jet growth and maximum-velocity decay ($Re_S = 27000$).

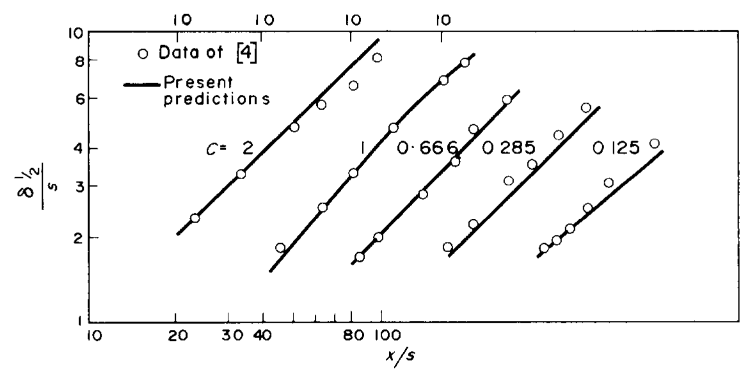

FIG. 3. Growth of the wall jet.

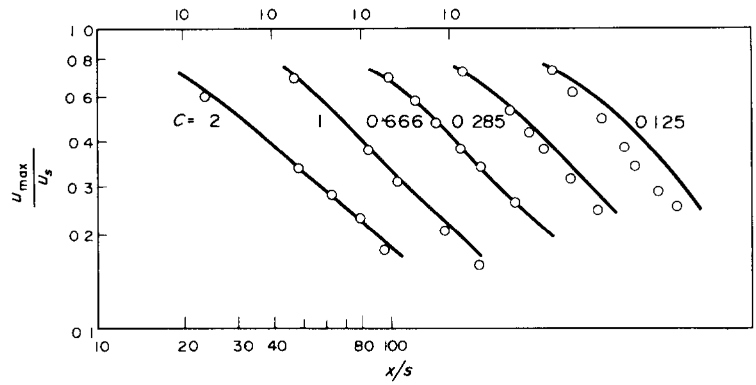

FIG. 4. Decay of the maximum velocity.

heat-transfer measurements in cylindrical wall jets. The purpose of this communication is to compare the predictions based on the mixing-length hypothesis with the above-mentioned experimental data.

2. DETAILS OF THE PREDICTION PROCEDURE

2.1. *The equations solved*

The equations describing the velocity and temperature fields in a uniform-property axi-symmetric boundary layer are:

$$\text{continuity:} \quad \frac{\partial u}{\partial x} + \frac{1}{r}\frac{\partial}{\partial r}(rv) = 0, \tag{2.1}$$

$$\text{momentum:} \quad \rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial}{\partial r}(r\tau) - \frac{dp}{dx}, \tag{2.2}$$

$$\text{energy:} \quad \rho u \frac{\partial T}{\partial x} + \rho v \frac{\partial T}{\partial r} = -\frac{1}{rc_p}\frac{\partial}{\partial r}(rq); \tag{2.3}$$

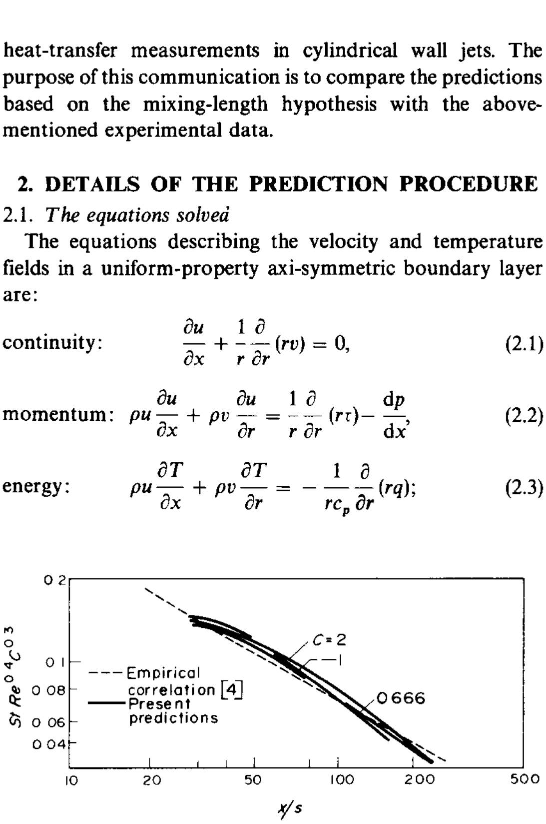

FIG. 5. Variation of the Stanton number.

the symbols have been defined in Nomenclature. The shear stress τ and the heat flux q are related to other quantities by a turbulence model, the details of which now follow.

2.2. *The turbulence model*

The exchange laws. The physical hypotheses used here are identical to those in [1], where more information can be found. The quantities τ and q are expressed as:

$$\tau = \mu_{eff}(\partial u/\partial r), \tag{2.4}$$

$$q = -(\mu_{eff}/\sigma_{eff})(c_p \partial T/\partial r), \tag{2.5}$$

where μ_{eff} is the effective (laminar + turbulent) viscosity and σ_{eff} is the effective Prandtl number.

The effective viscosity. The Prandtl mixing-length hypothesis gives:

$$\mu_{eff} = \rho l^2 |\partial u/\partial r|, \tag{2.6}$$

where l is the so-called mixing length. We use the following variation of the mixing length:

$$\begin{aligned} 0 < (r - r_I) \leqslant \lambda\delta/K &: l = K(r - r_I) \\ (r - r_I) > \lambda\delta/K &: l = \lambda\delta. \end{aligned} \tag{2.7}$$

This variation has been found to agree with a large amount of experimental data, and has been used in [1] and [2]. K and λ are constants and δ is the boundary-layer thickness. The values used here are

$$K = 0{\cdot}435, \qquad \lambda = 0{\cdot}09$$

which are the same as the ones in [1] and [2].

Equation (2.6) is used for the "fully-turbulent" region of the boundary layer where the turbulent viscosity is much larger than the laminar one. Near the wall, the effective viscosity is calculated from the van Driest hypothesis. The details of this are given in [1]; here it will suffice to note that the van Driest formula is in the present problem equivalent to the use of the logarithmic law of the wall.

The effective Prandtl number. Here we assume the effective Prandtl number to be uniform and equal to 0·9. Very near the wall, the extra resistance of the sub-layer is calculated by the use of the van Driest hypothesis.

General remarks. It is not claimed here that the set of hypotheses used is the best one or truly describes the turbulent exchange. This set has been found to give satisfactory predictions in many situations. Our purpose here is to demonstrate that the *same* set can successfully predict the cylindrical wall jet.

2.3. *The solution procedure*

The equations in Section 2.1 were solved by an implicit, finite-difference procedure described in [1]. The procedure uses a grid that expands or contracts to fit the important region of the flow. The computations reported here used 20 grid points in the cross-stream direction; the forward step in the x direction was one-fourth of the boundary-layer thickness. The computations were performed on the IBM 7044 computer at I.I.T. Kanpur.

3. COMPARISON WITH EXPERIMENTAL DATA

3.1. *The flow characteristics*

Starr and Sparrow [3] have reported flow measurements in a cylindrical wall jet having a curvature parameter C ($\equiv$ rod diameter/slot height) of 5.9. Figure 2 shows the growth of the wall jet and the decay of the maximum velocity. Our predictions are seen to agree very well with the experimental data.

Manian *et al.* [4] have conducted a more extensive experimental study which covers a large range of the curvature parameter C. The jet growth and velocity decay are compared with our predictions in Figs. 3 and 4; the agreement is once again satisfactory.

3.2. *The heat-transfer characteristics*

Manian *et al.* [4] also give heat-transfer measurements for cylindrical wall jets. They used a uniform heat flux from the cylinder wall. They show that the experimental data can be correlated by the dashed line in Fig. 5. The full lines show our predictions for some curvature parameters. The agreement can be seen to be very good.

4. CONCLUDING REMARKS

(1) The implications of the mixing-length hypothesis agree well with available experimental data for jet spread, velocity decay and heat transfer in cylindrical wall jets.

(2) It is emphasised here that the agreement mentioned above is *not* obtained by *adjusting* the constants in the hypothesis to fit the data. The physical hypotheses and the values of the constants are exactly the same as used in [1] and [2] for more conventional geometries.

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HEAT FLOW INTO A SEMI-INFINITE BODY WITH SURFACE TEMPERATURE A NON-INTEGER POWER OF TIME

D. B. R. KENNING*

Department of Mechanical Engineering, Massachusetts Institute of Technology, Cambridge, Massachusetts, U.S.A.

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NOMENCLATURE

A, constant defined in equation (6);
B. constant defined in equation (7);
k. thermal conductivity;
q_t, surface heat flux at time t;
Q_t, total surface heat flow from time 0 to t;
t, time;
x, distance from surface;
α, thermal diffusivity;
Θ, temperature.

* Permanent address, Department of Engineering Science, Oxford University, Parks Road, Oxford.